

# Adiabatic quantum pumping in an Aharonov-Bohm loop and in a Si-like nanowire: Role of interference in real space and in momentum space

Sungjun Kim,<sup>1</sup> Kunal K. Das,<sup>2</sup> and Ari Mizel<sup>1</sup>

<sup>1</sup> Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

<sup>2</sup> Department of Physics, Fordham University, Bronx, New York 10458, USA

We study the consequences of interference effects on the current generated by adiabatic quantum pumping in two distinct one-dimensional (1D) lattice model. The first model contains an Aharonov-Bohm (AB) loop within a tight-binding chain of lattice sites. The static AB phase is shown to strongly affect interference between the two arms of the loop, serving as an on-off switch and regulator for the pumped current. The second model simulates pumping in semiconductors with indirect band-gaps, by utilizing a tight-binding chain with next-nearest-neighbor coupling. The model exhibits signatures of interference between degenerate conduction band states with different Fermi wavevectors.

PACS numbers: 73.23.-b,73.63.-b,72.10.Bg,72.80.Cw

## I. INTRODUCTION

Adiabatic quantum pumping provides a mechanism to generate a direct current with no bias<sup>1</sup>. By periodically changing the parameters that define a conduction channel, one forces carriers down the channel. The quantity of pumped carriers depends only upon the path that the parameters take through parameter space and not upon the speed with which this path is traversed<sup>2</sup> (as long as the change is slow enough to remain adiabatic). Recently, adiabatic quantum pumping has been applied to a number of different transport contexts, including the generation of spin polarized current<sup>3,4,5,6,7,8,9</sup> and entangled pairs<sup>10,11,12</sup>. Experiments continue to be motivated by sustained interest in this unusual mesoscopic transport mechanism<sup>5,13,14</sup>.

Since pumping itself is a consequence of quantum interference, it is of interest to see how it is affected by other quantum interference phenomena working in conjunction. In this work we study two pump configurations which create additional interference effects due to (i) differences in spatial trajectories, and (ii) due to competing momenta. It is natural to think of the former as an interference effect in position space and the latter as an interference effect in momentum space.

The first configuration comprises of an Aharonov-Bohm (AB) loop geometry, subject of many studies associated with AB phase effects and Fano effects<sup>15,16,17,18</sup>. A pair of parallel quantum dots straddling the linear chain serve as the two arms of an AB loop. Differences in spatial trajectories are created in the two arms of the loop. We find surprising sensitivity of the pumped charge to the presence and magnitude of a *static* magnetic field associated with the AB effect; even with two time varying and out of phase parameters, there is no pumped current when the field is absent.

Early experimental work on adiabatic quantum pumping<sup>19</sup> observed a pumped current that is symmetric under magnetic field reversal, while theory predicted no definite symmetry<sup>20</sup>. It was suggested that the ob-

served currents may be due to rectification effects rather than pumping<sup>21</sup>. It is thus desirable to have a model in which pumped current and rectification current can be definitively distinguished. Rectification currents should be symmetric under magnetic field reversal, our model is therefore designed to have a mirror symmetry that causes the pumped charge to be antisymmetric  $q_{\text{pump}}(-B) = -q_{\text{pump}}(B)$  under magnetic field reversal. Thus, experiments conducted on a realization of this model would have some natural explanations of an experimentally observed current: If the current is anti-symmetric under magnetic field reversal, it is likely due to quantum pumping, otherwise unexpected mechanism is involved in generating the resulting current.

The second model that we consider is a tight-binding chain with next-nearest-neighbor (nnn) hopping; in that case the minimum of the conduction band energy need not lie at  $k = 0$ ; and the conduction band can look like that of a semiconductor with an indirect gap. A physical motivation for this model is the band structure of Si nanowires. Calculations have predicted an indirect gap<sup>22</sup> when Si nanowires are grown along certain directions, say, the  $[11\bar{2}]$  direction. The conduction band dispersion relation in our model can have four Fermi wave vectors at a given Fermi energy  $E_F$ . Our calculations show rich interference effects between these wavevectors, including resonant peaks of different heights in the pumped current.

The rest of the paper is organized as follows. In Sec. II, we formulate our method for computing the current pumped in a one dimensional lattice due to an arbitrary localized time varying potential. In Sec. III, we compute the effects of a static magnetic field on the AB loop system and demonstrate the spatial interference effects induced by that field on the pumped current. We study the effects of nnn coupling in Sec. IV and analyze wavevector interference phenomena. We summarize our primary conclusions in Sec. V.

## II. MECHANISM OF ADIABATIC PUMPING IN AN ONE DIMENSIONAL CHAIN

The description of adiabatic pumping in mesoscopic systems is based upon the Landauer-Büttiker scattering picture. For a single channel in a system describable by a one-dimensional lattice Hamiltonian, the instantaneous pumped current is<sup>9,23</sup>

$$j_{\text{pump}}(n) = e \int dE f(E) \int_{-\pi}^{\pi} \frac{dp}{2\pi} \delta(E - E_p) \langle \phi_p | j_n | \phi_p \rangle, \quad (1)$$

where  $j_n$  is the appropriate discrete current operator for the one-dimensional chain. The system has a single Fermi distribution function  $f(E)$  since pumping operates in the absence of bias. The matrix elements of the current operator are taken with respect to scattering states  $|\phi_p(t)\rangle$  of the full *time-dependent* Hamiltonian  $H(t)$  of the system. When the time dependence of the Hamiltonian is adiabatic, the states  $|\phi_p(t)\rangle$  can be approximated by an expansion in terms of the *instantaneous* scattering states  $|\chi_p(t)\rangle$  up to linear order in the time derivative

$$|\phi_p(t)\rangle = |\chi_p(t)\rangle - i\hbar G(E_p) |\dot{\chi}_p(t)\rangle. \quad (2)$$

Here,  $G(E_p) = 1/(E_p - H + i\eta)$  is the retarded Green's function for the *instantaneous* Hamiltonian  $H$  with  $\eta = 0^+$  imposing causality. The instantaneous scattering states

$$|\chi_p(t)\rangle = (1 + G(E_p)V)|p\rangle, \quad (3)$$

are exact solutions to the Lippmann-Schwinger equation for the instantaneous scattering potential  $V$  that contains the time-varying part of the full Hamiltonian  $H$  with  $|p\rangle$  being a plane wave state. If the time dependence in  $H$  is not explicit but instead arises only through time-dependent parameters, the states  $|\phi_p(t)\rangle$  and  $|\chi_p(t)\rangle$  inherit time dependence through those parameters and also do not explicitly depend on time.

We apply the above considerations to a 1D-lattice Hamiltonian comprising of the standard time-independent tight-binding Hamiltonian  $H_0$  with nearest-neighbor coupling and a general time-dependent potential  $V(t)$  acting on a finite 'scattering region' of the lattice:

$$\begin{aligned} H(t) &= H_0 + V(t), \\ H_0 &= -J \sum_n (a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1}), \\ V(t) &= \sum_{x,y} V_{xy}(t) a_x^\dagger a_y, \end{aligned} \quad (4)$$

Here  $a_n^\dagger$  is the electron creation operator at site  $n$  and  $-J$  is the nearest neighbor hopping coupling strength. The on-site energy of the sites of the free Hamiltonian  $H_0$  is taken to be the reference energy and hence set to zero. The potential  $V(t)$  is parameterized by the  $V_{xy}(t)$  which contribute to the inter-site coupling strengths ( $x \neq y$ )

and on-site energy shifts ( $x = y$ ) within the scattering region. Since the current is always defined asymptotically far from the scattering region,  $H_0$  determines the definition of the current operator

$$j_n = -\frac{J}{i\hbar} (a_{n+1}^\dagger a_n - a_n^\dagger a_{n+1}) \quad (5)$$

as well as the dispersion relation  $E_p = -2J \cos p$ , where the spacing between the sites in the chain has been set to unity, thereby setting a natural length scale.

We evaluate the pumped current for this Hamiltonian in the distant region  $n \rightarrow \infty$  through an adaptation of an analysis we presented in a recent publication<sup>9</sup>. We only consider real pumping parameters in this paper, so we set  $\dot{V}_{xy} = \dot{V}_{yx}$  for all  $x, y$ . The pumped current for the time-dependent Hamiltonian in Eq. (4) is then given by

$$\begin{aligned} j_{\text{pump}}(n) &= -\frac{eJ}{\pi} \int dE f(E) \times \\ &\quad \text{Im} \sum_{x,y} \dot{V}_{xy} \partial_E [G^*(E)(n+1, x) G(E)(n, y)], \end{aligned} \quad (6)$$

We used the notation  $G(E)(n, x) = \langle 0 | a_n G(E) a_x^\dagger | 0 \rangle$ . Since pumping experiments require low temperatures we take the zero temperature limit whereby  $f(E)$  becomes a step function and an integration by parts transforms Eq. (6) for the pumped current to

$$j_{\text{pump}}(n) = -\frac{eJ}{\pi} \text{Im} \sum_{x,y} \dot{V}_{xy} G^*(E_F)(n+1, x) G(E_F)(n, y), \quad (7)$$

where  $G(E_F)(n, x)$  is the full propagator in the energy domain evaluated at the Fermi energy. We will take our time-varying parameters to be on-site energies, so only the diagonal elements of  $V_{xy}$  will have non-vanishing time-derivatives and we simplify the notation by defining  $\dot{V}_{xy} = \dot{u}_x \delta_{x,y}$ . Using Dyson's equation<sup>24</sup>,  $G(E) = G_0(E) + G_0(E) V G(E)$ , where  $G_0(E) = 1/(E - H_0 + i\eta)$ , we can show that  $G(E_F)(n, x) = -2\pi i N(E_F) [e^{ik_F(n-x)} + \sum_{x'} u_{x'} e^{ik_F(n-x')} G(E_F)(x', x)]$  for  $n$  larger than all  $x, x'$  in the scattering region. Thus we have the identity  $G(E_F)(n+1, x) = e^{ik_F} G(E_F)(n, x)$  for  $n$  larger than all  $x$  in the scattering region, and the expression (7) for the pumped current becomes

$$j_{\text{pump}}(n) = \frac{e}{(2\pi)^2} N(E_F)^{-1} \sum_x \dot{u}_x |G(E_F)(n, x)|^2. \quad (8)$$

with  $N(E_F) = \frac{1}{2\pi} \frac{\partial k}{\partial E_k} \big|_{k=k_F}$  being the one-dimensional density of states per unit length. In fact, using the Dyson equation result, one can show that the current is independent of the site index  $n$  as one should expect from charge conservation

$$\begin{aligned} j_{\text{pump}} &= e N(E_F) \sum_x \dot{u}_x \times \\ &\quad \left| 1 + \sum_{x'} u_{x'} e^{-ik_F(x'-x)} G(E_F)(x', x) \right|^2. \end{aligned} \quad (9)$$

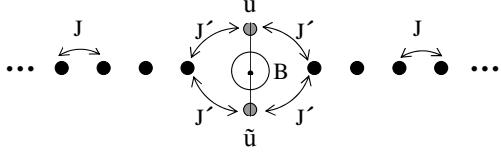


FIG. 1: Two quantum dots are connected to left and right leads in parallel. They form a closed loop in the central region which is threaded by a magnetic field  $B$ . One dot has on-site energy  $u$  and the other has on-site energy  $\tilde{u}$ . The symbols  $-J$  and  $-J'$  are hopping amplitudes. The vertical line shows the plane of left-right reflection symmetry when  $B = 0$ .

In this expression, the instantaneous pumped current is determined by the density of states, instantaneous potentials and their time derivatives, and the *local* full propagator (local since  $x$  and  $x'$  are both in the scattering region where  $u_x$  and  $u_{x'}$  are both non-zero), which can be determined from Dyson's equation from a knowledge of the potential and the free Green's function.

### III. PUMPING THROUGH A LOOP GEOMETRY

We now apply the considerations of the previous section to determine the pumped current for our first physical model of interest, consisting of a 1-D tight-binding chain with a loop in the central region as shown in Fig. 1. The Hamiltonian has the general form shown in Eq (4) where potential is now specifically

$$V(t) = u(t)b_0^\dagger b_0 + \tilde{u}(t)\tilde{b}_0^\dagger \tilde{b}_0 + J[a_0^\dagger a_{-1} + a_1^\dagger a_0] - J'e^{i\varphi/4}[\tilde{b}_0^\dagger a_{-1} + a_1^\dagger \tilde{b}_0] - J'e^{-i\varphi/4}[b_0^\dagger a_{-1} + a_1^\dagger b_0] + (\text{Hermitean conjugate}). \quad (10)$$

To obtain this potential, we imagine displacing site 0 above the chain so that it forms the upper arm of a loop. We define the creation operator  $b_0^\dagger \equiv a_0^\dagger$ , introducing new notation for the displaced state. We introduce a new site to form the lower arm; the creation operator associated with this new site is  $\tilde{b}_0^\dagger$ . Thus, the two parallel sites in the middle of the chain are represented by  $b_0^\dagger$  and  $\tilde{b}_0^\dagger$ . These sites are decoupled from each other but are coupled to the rest of the chain on either side with a coupling strength of  $-J'$ . The time-dependence lies at the on-site energies of the two parallel sites  $\tilde{u}$  and  $u$ , which serve as the pumping parameters. The two sites straddling the chain create a loop, and a static magnetic flux  $\Phi$  penetrating that loop creates an Aharonov-Bohm phase difference  $\varphi = 2\pi\Phi/\Phi_0$ , where  $\Phi_0 = hc/e$ , between the two spatial paths defined by the arms of the loop, leading to interference effects.

By specifying a cyclic time-dependence of the parameters  $u$  and  $\tilde{u}$  and the definition of the potential, we can

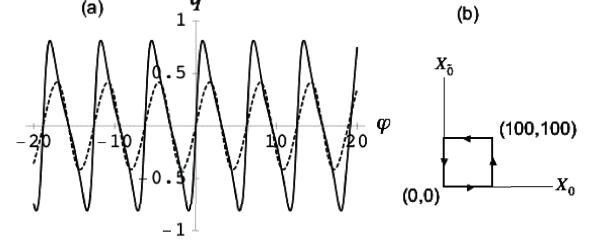


FIG. 2: (a) Charge  $q$  (in units of  $e$  throughout paper) pumped per cycle vs  $\varphi$  (in radians): We set  $J'/J = 0.5$  and traverse a square-shaped pumping cycle with corners  $(X_0, X_{\tilde{0}}) = (u/J, \tilde{u}/J) = (0,0)$  and  $(100,100)$  as shown in (b). The solid line in (a) is for  $k_F = 1.7$ , and the dashed line is for  $k_F = 2.1$ . This plot shows the antisymmetry and periodicity of pumped charge as a function of the magnetic field. The solid line curve near the origin shows that the pumped current is highly sensitive to small magnetic fields.

integrate the expression in Eq. (9) to compute the charge pumped in a full cycle

$$q_{\text{pump}} = \oint dt j_{\text{pump}}(n). \quad (11)$$

We choose a simple square-loop time cycle as shown in Fig. 2(b) in the space of the two pumping parameters. The charge pumped over a cycle is computed numerically and the behavior as a function of the Aharonov-Bohm phase  $\varphi$  is presented in Fig. 2(a). The dependence as a function of the Fermi vector  $k_F$  is shown in Fig 3. The results show several interesting features that we now discuss in detail.

(i) The current vanishes in the absence of the static magnetic field. The magnetic field plays a crucial role even though it does not vary in time and therefore never acts as a pumping parameter. The necessity for the static magnetic field can be understood by recognizing that our system has reflection symmetry about a plane through the two loop sites, orthogonal to the direction of flow. It is easy to see by simply reversing the magnetic field, and simultaneously exchanging sites  $n$  with  $-n$ , that the reflection symmetry implies

$$\oint dt j_{\text{pump}}(n, -\varphi) = - \oint dt j_{\text{pump}}(n, \varphi) \quad (12)$$

This is an example of a discrete symmetry<sup>20</sup> which causes the pumped charge to be antisymmetric under reversal of magnetic field. This antisymmetry is also manifest in Fig. 3, where the two traces of the pumped charge, as a function of the Fermi vector  $k_F$ , are exactly antisymmetric because one is for  $\varphi = \pi/2$  and the other is for  $\varphi = 3\pi/2 \equiv -\pi/2$ .

More intuitively, when the magnetic field is absent, the two parallel quantum dots effectively pump at the same location along the current flow. The two potentials combine into a single parameter, and a single parameter cannot pump any current. The static magnetic field

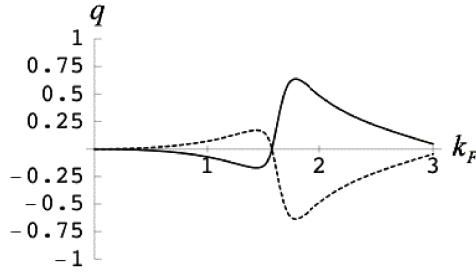


FIG. 3: Pumped charge  $q$  vs  $k_F$ : We set  $J'/J = 0.5$  and traverse a square-shaped pumping cycle with corners  $(X_0, X_0) = (0, 0)$  and  $(100, 100)$ . The solid line is for  $\varphi = \frac{\pi}{2}$ , which corresponds to  $\Phi = \frac{1}{4}\Phi_0$ , and the dashed line is for  $\varphi = \frac{3\pi}{2}$ , which corresponds to  $\Phi = \frac{3}{4}\Phi_0$ . Since the current is periodic for  $\varphi$  and is antisymmetric when the sign of  $\varphi$  is reversed, the two curves shown have opposite values of pumped charge at any given  $k_F$ .

breaks the reflection symmetry, and creates a phase shift between the two dots. They no longer lie symmetrically at the same location in the current flow, and now act as two separate pumping parameters, producing a current. In Ref. 25, a related effect was seen, in a very different configuration involving two loops where the pumping parameters were time-varying magnetic fields. In that work, the static field also breaks the symmetry, allowing a pumping current to arise.

(ii) There is a periodicity in the charge pumped with respect to the phase introduced by the static magnetic field, as shown in Fig. 2. This periodicity is an expected feature of AB phenomena. The periodicity of  $\varphi = 2\pi$  causes the pumped charge to vanish when the phase difference due to the magnetic field between the two arms of the loop vanishes, equivalent to having no field at all.

(iii) The pumped charge also vanishes for certain values of  $k_F$  specifically at  $k_F = n\pi/2$ . This can be seen in Fig. 3, where the pumped charge is plotted as a function of  $k_F$  for two different values of the Aharonov-Bohm phase  $\varphi$ . The reason is that an electron picks up a phase of  $\pm k_F$  as it moves from site to adjacent site. Therefore in traversing one of the arms of the loop it picks up a total phase of  $\pm 2k_F \pm \varphi/2$ , and in going through the other arm, the corresponding total phase change is  $\mp 2k_F \mp \varphi/2$ . When  $k_F = \pi/2$ , they become respectively  $\pm \pi \pm \varphi/2$  and  $\mp \pi \mp \varphi/2 \equiv 2\pi - (\mp \pi \pm \varphi/2) \equiv \pm \pi \pm \varphi/2$ . This shows that the phases accumulated in both paths are identical and therefore the symmetry-breaking effect of the magnetic field is annulled and the pumped charge vanishes as if the magnetic field was not there at all.

We conclude the discussion of this system by noting that the pumped current is quite sensitive to changes in the external magnetic field as seen from the solid curve in Fig. 2(a). This can provide a way of precisely controlling the magnitude and direction of the pumped current without changing the time-varying pumping parameters in any way. In addition the fact that the absence of the

field leads to vanishing current suggests an obvious application as a switching mechanism.

#### IV. PUMPING ON A CHAIN WITH NEXT-NEAREST-NEIGHBOR HOPPING

We now turn our attention to the pumped current on a chain with next-nearest-neighbor (nnn) hopping (see Fig. 4). We consider the following Hamiltonian

$$\begin{aligned} H(t) &= H_0 + V(t), \\ H_0 &= -J \sum_n (a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1}) \\ &\quad - J' \sum_n (a_{n+2}^\dagger a_n + a_n^\dagger a_{n+2}), \\ V(t) &= u_{-l}(t) n_{-l} + u_l(t) n_l, \end{aligned} \quad (13)$$

where  $-J'$  is the nnn hopping amplitude and  $n_l = a_l^\dagger a_l$  is the number operator on site  $l$ . The on-site energy is taken to be zero for all sites except for the sites  $\pm l$ . The energies at those sites  $u_l$  and  $u_{-l}$  are the time varying pumping parameters.

The dispersion relation for this model (taking the lattice constant to be unity) is  $E_k = -2J \cos k - 2J' \cos 2k$ . We assume a positive  $J$  and negative  $J'$  so that the dispersion relation yields a double-well shape as shown in Fig. 5. This “indirect gap” shape is physically relevant, being reminiscent, for instance, of the band structure of certain Si nanowires<sup>22</sup>. For any total energy  $E < 0$  in Fig. 5, solving  $E_k = E$  yields four solutions  $\{\pm k_1(E), \pm k_2(E)\}$ , related by  $\cos k_1 + \cos k_2 = -J/(2J')$ . At zero temperature the pumping dynamics is determined by the Fermi energy  $E = E_F$ ; we denote the corresponding wavevectors  $\{\pm k_{1F}, \pm k_{2F}\}$ . We choose the convention that  $k_1 > k_2 > 0$ .

We need to extend the analysis of Sec. II to get the pumped current by defining an appropriate discrete current operator for this extended chain. Since there is a nnn hopping process, we define the current operator using the continuity equation  $\partial_t [\rho(n) + \rho(n+1)] + J(n+1) - J(n-1) = 0$ , where  $\rho(n) = \langle \psi | a_n^\dagger a_n | \psi \rangle$  and  $J(n) = \langle \psi | J_n | \psi \rangle$ .

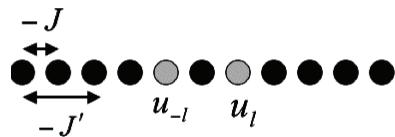


FIG. 4: We consider a 1D tight-binding chain with both nearest neighbor and next-nearest-neighbor coupling, where  $-J$  is the strength for nearest neighbor hopping, and  $-J'$  is the strength for next-nearest-neighbor hopping. On this chain, we assume only two sites (gray dots) located at  $-l$  and  $l$  have non-zero on-site energies  $u_{-l}, u_l$ .

We are led to the definition of the operator

$$\begin{aligned} J_n &= -\frac{J}{i\hbar}(a_{n+1}^\dagger a_n - a_n^\dagger a_{n+1}) - \frac{J'}{i\hbar}(a_{n+1}^\dagger a_{n-1} - a_{n-1}^\dagger a_{n+1}) \\ &\quad - \frac{J'}{i\hbar}(a_{n+2}^\dagger a_n - a_n^\dagger a_{n+2}). \end{aligned} \quad (14)$$

We compute the pumped current using this current operator in the expression for the current (1), instead of  $j_n$ ; (No confusion should arise between the current operator  $J_n$  and the tunneling parameters  $J$  and  $J'$ .) In the zero temperature limit and at points far away ( $n \rightarrow \infty$ ) from the action of the pumping potential, the pumped current is

$$\begin{aligned} J_{pump}(n) &= e \int dE f(E) \int_{-\pi}^{\pi} \frac{dp}{2\pi} \delta(E - E_p) \langle \phi_p | J_n | \phi_p \rangle \quad (15) \\ &= -\frac{eJ}{\pi} \text{Im} \sum_{x=\pm l} \dot{u}_x G(E_F)(n, x) G^*(E_F)(n+1, x) \\ &\quad - \frac{eJ'}{\pi} \text{Im} \sum_{x=\pm l} \dot{u}_x G(E_F)(n-1, x) G^*(E_F)(n+1, x) \\ &\quad - \frac{eJ'}{\pi} \text{Im} \sum_{x=\pm l} \dot{u}_x G(E_F)(n, x) G^*(E_F)(n+2, x). \end{aligned}$$

Some care is needed in applying Eq. (15). Because of the shape of the dispersion relation, the velocity  $v_k = \frac{1}{\hbar} \frac{\partial E_k}{\partial k}$  for  $+k_2$  is negative while the velocity for  $-k_2$  is positive. Thus incoming waves from the left reservoir corre-

spond to wave vectors  $\{k_1, -k_2\}$  rather than  $\{k_1, k_2\}$ . To keep this straight, when computing the full Green's function  $G(E) = 1/(E - H + i\eta)$ , we use  $\eta = \eta_1 = 0^+$  at the  $k_1$  singularity in the denominator and  $\eta = \eta_2 = 0^-$  at the  $k_2$  singularity in the denominator. As a result, left incoming waves with  $k_1$  or  $-k_2$  have transmitted waves with the correct physical wave vectors  $\{k_1, -k_2\}$  and reflected waves with the correct physical wave vectors  $\{-k_1, k_2\}$ .

Using Dyson's equation to compute the full Green's function, we derive an explicit expression for the instantaneous pumped current

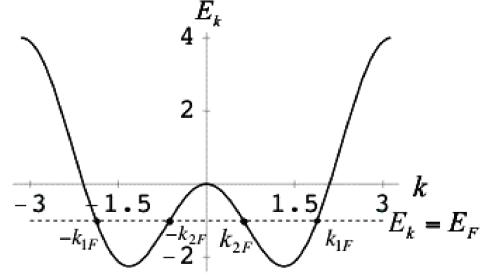


FIG. 5: The dispersion relation for  $J = 1$  and  $J' = -1$ . It has a double-well shape. The dashed line is for  $E_k = E_F = -1$ ; it has four crossing points with the doublewell curve, corresponding to four Fermi wave vectors  $\{k_{1F}, k_{2F}, -k_{1F}, -k_{2F}\}$ .

$$\begin{aligned} J_{pump} &= -\frac{2eJ'}{\pi} \sum_{x=\pm l} \dot{u}_x [g_1^2 |d(-x) + e^{2ik_1 x} h(-x)|^2 \sin k_1 (\cos k_1 - \cos k_2) \\ &\quad + g_2^2 |d(-x) + e^{-2ik_2 x} h(-x)|^2 \sin k_2 (\cos k_1 - \cos k_2)] \Big|_{E=E_F}, \end{aligned} \quad (16)$$

where

$$d(x) = \frac{1 - u_x G_0(E)(0, 0)}{Z_x}, \quad (17a)$$

$$h(x) = \frac{u_x G_0(E)(x, -x)}{Z_x}, \quad (17b)$$

$$Z_x = [1 - u_x G_0(E)(0, 0)][1 - u_{-x} G_0(E)(0, 0)] - u_{-x} u_x G_0^2(E)(x, -x), \quad (17c)$$

$$g_1 = \frac{1}{2iJ \sin k_1 + 4iJ' \sin 2k_1}, \quad (17d)$$

$$g_2 = -\frac{1}{2iJ \sin k_2 + 4iJ' \sin 2k_2}, \quad (17e)$$

$$G_0(E)(x, y) = g_1 e^{ik_1 |x-y|} + g_2 e^{-ik_2 |x-y|}. \quad (17f)$$

The free Green's function  $G_0(E)(x, y)$  is found to be the sum of two terms  $g_1 e^{ik_1 |x-y|}$  and  $g_2 e^{-ik_2 |x-y|}$ . As a result, interference effects arise between the two wave vectors  $\{k_1, -k_2\}$ . In particular, the  $G_0^2(x, -x)$  term in  $Z_x$ , defined above, contains a factor of the form

$e^{2i(k_1 - k_2)|x|}$  which depends on the sum of the two wave vectors,  $k_1 - k_2$  and the distance  $2|x|$  between the locations of the two time varying sites  $n_l$  and  $n_{-l}$  in the potential  $V(t)$ . This factor of  $e^{2i(k_1 - k_2)|x|}$  admits an interpretation in terms of interference between the wavefunction with vector  $k_1$  and the one with vector  $-k_2$ . Physically speaking, the dependence of the pumped current on the separation  $2|x| = 2l$  between the two points of action of the potentials in  $V$  depends not only on  $k_1$  and  $-k_2$  individually but also on  $k_1 - k_2$ . This leads to a rather irregular pattern for the pumped current as function of the separation  $2l$  as seen in Fig. 6(b). This is quite distinct from the regular pattern, shown in Fig. 6(a) for comparison, for standard nearest neighbor coupling i.e. when  $J' = 0$ . In all cases  $u_l$  and  $u_{-l}$  trace out a square-shaped time cycle like the one shown in Fig. 2

In Fig. 7, we investigate how the pumped current depends upon the Fermi wave vector  $k_{1F}$ ; note that  $k_{1F}$  determines the Fermi energy  $E_F$ , via the dispersion re-

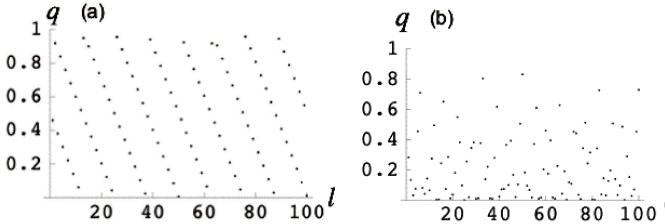


FIG. 6: Pumped charge  $q$  vs  $l$  (there is exactly one value of  $q$  for each choice of  $l$  in both (a) and (b)). In both (a) and (b), the Fermi level is  $E_F = -1.5$ , and the pumping cycle is square shaped with lower-left corner  $(0, 0)$  and upper-right corner  $(100, 100)$ . The distance between the two time-dependent potentials is  $2l$ . (a) Standard chain with  $J = 1$  and no nnn hopping,  $J' = 0$ . A regular periodicity is evident. (b) Chain with nnn hopping,  $J = 1$  and  $J' = -1$ . The dependence of  $q$  on  $l$  is quite irregular with no obvious pattern.

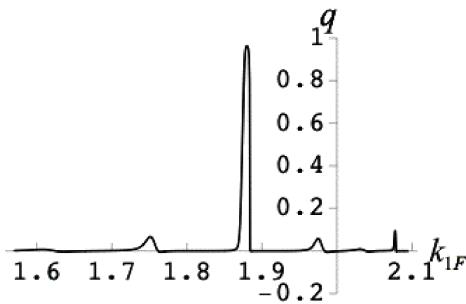


FIG. 7: Pumped charge  $q$  vs  $k_{1F}$ : We set  $J = 1$ ,  $J' = -1$  and  $l = 10$ , and use a square-shaped pumping cycle with left-lower corner at  $(4, 4)$  and right-upper corner at  $(100, 100)$ . High peak near  $k_{1F} = 1.9$  is related to resonance transmission and low peak between  $k_{1F} = 1.7$  and  $k_{1F} = 1.8$  indicates destructive interference between two wave vectors.

lation, as well as  $-k_{2F}$ . The high peak near  $k_{1F} = 1.9$  results from a resonance effect, as we now explain. Note that the transmission will be greatest when the denominator  $Z_l$  in Eq. (17c) is as small as possible. For large  $u_l, u_{-l}$ , this means minimizing the terms in  $Z_l$  that are multiplied by the product  $u_l u_{-l}$ . This leads to the condition  $G_0^2(E)(l, -l) - G_0^2(E)(0, 0) = 0$ , which corre-

sponds to three resonance conditions for the wave vectors:  $e^{4ik_{1F}l} = 1$ ,  $e^{-4ik_{2F}l} = 1$ , and  $e^{2i(k_{1F} - k_{2F})l} = 1$ . Now when the Fermi wave vector is close to  $k_{1F} \simeq 1.9$ , where the strong peak occurs in Fig. 7, all three conditions are satisfied and a large pumped charge  $q \simeq 1$  arises. In case of the smaller peak between  $k_{1F} = 1.7$  and  $k_{1F} = 1.8$  in Fig. 7, the value of  $k_{1F}$  satisfies the first two resonance conditions but not the third. Physically, this can be interpreted as destructive interference between the two wave vectors, resulting in a smaller pumped charge.

## V. CONCLUSION

We have used two distinct physically relevant models to illustrate how the charge pumped in an adiabatic pumping process can be strongly influenced by interference effects in both position space and momentum space. Specifically, we have demonstrated that pumped current can be generated using a single Aharonov-Bohm loop provided a static magnetic field is present. Although that field itself is not an active pumping parameter, the pumped charge is very sensitive to its magnitude and direction, a feature that can be used for very delicate control of the pumping process. As we noted, the fact that the pumped current is antisymmetric in the magnetic field suggests an experimental means of clearly distinguishing pumped current and current arising in another way that lacks this antisymmetry. Using a next-nearest-neighbor-coupling lattice model to simulate the indirect-gap present in the band-structure of Si, we demonstrated that interference effects in momentum space cause strong and weak resonant peaks in the pumped current per cycle. The models considered here demonstrate that interference phenomena provide powerful techniques for altering adiabatic pumping behavior.

## Acknowledgments

We gratefully acknowledge the support of the Packard Foundation, NSF NIRT program Grant No. DMR-0103068 and the Research Corporation.

---

<sup>1</sup> P. W. Brouwer, Phys. Rev. B **58**, R10 135 (1998).  
<sup>2</sup> B. L. Altshuler and L. I. Glazman, Science **283**, 1864 (1999).  
<sup>3</sup> P. Sharma and C. Chamon, Phys. Rev. Lett. **87**, 096401 (2001).  
<sup>4</sup> E. R. Mucciolo, C. Chamon, and C. M. Marcus, Phys. Rev. Lett. **89**, 146802 (2002).  
<sup>5</sup> S. K. Watson, R. M. Potok, C. M. Marcus, and V. Umansky, Phys. Rev. Lett. **91**, 258301 (2003).  
<sup>6</sup> T. Aono, Phys. Rev. B **67**, 155303 (2003).  
<sup>7</sup> Y. Wei, L. Wan, B. Wang, and J. Wang, Phys. Rev. B **70**, 045418 (2004).

<sup>8</sup> M. Blaauboer and C. M. L. Fricot, Phys. Rev. B **71**, 041303(R) (2005).  
<sup>9</sup> S. Kim, K. K. Das, and A. Mizel, Phys. Rev. B **73**, 075308 (2006).  
<sup>10</sup> C. W. J. Beenakker, M. Titov, and B. Trauzettel, Phys. Rev. Lett. **94**, 186804 (2005).  
<sup>11</sup> P. Samuelsson and M. Büttiker, Phys. Rev. B **71**, 245317 (2005).  
<sup>12</sup> K. K. Das, S. Kim, and A. Mizel, Phys. Rev. Lett. **97**, 096602 (2006); K. K. Das, S. Kim, and A. Mizel, Bull. Am. Phys. Soc. **49**, 218 (2004).  
<sup>13</sup> L. DiCarlo, C. M. Marcus, and J. S. Harris, Jr., Phys. Rev.

Lett. **91**, 246804 (2003).  
<sup>14</sup> M R Buitelaar, P J Leek, V I Talyanskii, C G Smith, D Anderson, G A C Jones, J Wei and D H Cobden, Semicond. Sci. Technol. **21**, S69 (2006).  
<sup>15</sup> Y. Gefen, Y. Imry, and M. Ya. Azbel, Phys. Rev. Lett. **52**, 129 (1984).  
<sup>16</sup> K. Kobayashi, H. Aikawa, S. Katsumoto, and Y. Iye, Phys. Rev. Lett. **88**, 256806 (2002).  
<sup>17</sup> A. Aharony, O. Entin-Wohlman, and Y. Imry, Phys. Rev. Lett. **90**, 156802 (2003).  
<sup>18</sup> H. Lu, R. Lü, and B.-f. Zhu, Phys. Rev. B **71**, 235320 (2005).  
<sup>19</sup> M. Switkes, C. M. Marcus, K. Campman, and A. C. Gossard, Science **283**, 1905 (1999).  
<sup>20</sup> I. L. Aleiner, B. L. Altshuler, and A. Kamenev, Phys. Rev. B **62**, 10373 (2000).  
<sup>21</sup> P. W. Brouwer, Phys. Rev. B **63**, 121303(R) (2001).  
<sup>22</sup> H. Scheel, S. Reich, and C. Thomsen, Phys. Stat. Sol. (b) **242**, 2474 (2005).  
<sup>23</sup> O. Entin-Wohlman, A. Aharony, and Y. Levinson, Phys. Rev. B **65**, 195411 (2002).  
<sup>24</sup> E. N. Economou, *Green's Functions in Quantum Physics*, (Springer-Verlag, Berlin, 1979).  
<sup>25</sup> Dongkwan Shin and Jongbae Hong, Phys. Rev. B **70**, 073301 (2004).